

Intermediate Algebra
Skill-Builder # AE - 9
Expanding Binomials Using Pascal's Triangle

$$\begin{aligned}
 (a+b)^0 &: && 1 \\
 (a+b)^1 &: && 1 & 1 \\
 (a+b)^2 &: && 1 & 2 & 1 \\
 (a+b)^3 &: && 1 & 3 & 3 & 1 \\
 (a+b)^4 &: && 1 & 4 & 6 & 4 & 1 \\
 (a+b)^5 &: & 1 & 5 & 10 & 10 & 5 & 1 \\
 (a+b)^6 &: & 1 & 6 & 15 & 20 & 15 & 6 & 1 \\
 (a+b)^7 &: & 1 & 7 & 21 & 35 & 35 & 21 & 7 & 1
 \end{aligned}$$

Examples Expand using Pascal's Triangle.

1. $(x+2)^5$

Solution:

The expansion will have **6 terms** with **coefficients 1, 5, 10, 10, 5, 1**. The terms will be in **descending powers of x** starting from x^5 and ending in x^0 or 1 and **ascending powers of 2** starting from 2^0 or 1 and ending in 2^5 . Thus,

$$(x+2)^5 = 1 \cdot x^5 \cdot 2^0 + 5 \cdot x^4 \cdot 2^1 + 10 \cdot x^3 \cdot 2^2 + 10 \cdot x^2 \cdot 2^3 + 5 \cdot x^1 \cdot 2^4 + 1 \cdot x^0 \cdot 2^5$$

and simplifying we get

$$(x+2)^5 = x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32.$$

2. $(3x-2y)^6$

Solution:

The expansion will have **7 terms** with **coefficients 1, 6, 15, 20, 15, 6, 1**. The terms will be in descending powers of $3x$ starting from $(3x)^6$ and ending in $(3x)^0$ or 1 and ascending powers of $-2y$ starting from $(-2y)^0$ or 1 and ending in $(-2y)^6$. Thus,

$$\begin{aligned}
 (3x-2y)^6 &= 1 \cdot (3x)^6 (-2y)^0 + 6 \cdot (3x)^5 (-2y)^1 + 15 \cdot (3x)^4 (-2y)^2 + 20 \cdot (3x)^3 (-2y)^3 + 15 \cdot (3x)^2 (-2y)^4 \\
 &\quad + 6 \cdot (3x)^1 (-2y)^5 + 1 \cdot (3x)^0 (-2y)^6
 \end{aligned}$$

Simplifying, we get

$$(3x-2y)^6 = 729x^6 - 2916x^5y + 4860x^4y^2 - 4320x^3y^3 + 2160x^2y^4 - 576xy^5 + 64y^6$$

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Expand using Pascal's Triangle.

1. $(x + y)^3$

2. $(x - 4)^3$

3. $(y + 4)^4$

4. $(n - 2)^4$

5. $(a + 3)^5$

6. $(x - 2)^6$

7. $(3a + 2b)^5$

8. $(2c - 3d)^6$

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Answers

1. $x^3 + 3x^2y + 3xy^2 + y^3$

2. $x^3 - 12x^2 + 48x - 64$

3. $y^4 + 16y^3 + 96y^2 + 256y + 256$

4. $n^4 - 8n^3 + 24n^2 - 32n + 16$

5. $a^5 + 15a^4 + 90a^3 + 270a^2 + 405a + 243$

6. $x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$

7. $243a^5 + 810a^4b + 1080a^3b^2 + 720a^2b^3 + 240ab^4 + 32b^5$

8. $64c^6 - 576c^5d + 2160c^4d^2 - 4320c^3d^3 + 4860c^2d^4 - 2916cd^5 + 729d^6$

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