

Intermediate Algebra
Skill Builder # PF – 14
Factoring by Substitution

We can use the ***substitution method*** to reduce a polynomial to a form we can easily recognize and factor.

Examples

1. $36x^4y^8 - 49z^{12}w^6$

Solution:

Note that each term is a perfect square:

$$36x^4y^8 - 49z^{12}w^6 = (6x^2y^4)^2 - (7z^6w^3)^2$$

If we let $a = 6x^2y^4$ and $b = 7z^6w^3$, we then get

$$\begin{aligned} 36x^4y^8 - 49z^{12}w^6 &= (6x^2y^4)^2 - (7z^6w^3)^2 \\ &= a^2 - b^2 \\ &= (a - b)(a + b) \\ &= (6x^2y^4 - 7z^6w^3)(6x^2y^4 + 7z^6w^3) \end{aligned}$$

2. $8a^6b^3 + 27c^{24}$

Solution:

Note that each term is a perfect cube:

$$8a^6b^3 + 27c^{24} = (2a^2b)^3 + (3c^8)^3$$

If we let $x = 2a^2b$ and $y = 3c^8$, we then get

$$\begin{aligned} 8a^6b^3 + 27c^{24} &= (2a^2b)^3 + (3c^8)^3 \\ &= x^3 + y^3 \\ &= (x + y)(x^2 - xy + y^2) \\ &= (2a^2b + 3c^8)(4a^4b^2 - 6a^2bc^8 + 9c^{16}) \end{aligned}$$

3. $x^4 - 6x^2 - 27$

Solution:

If we let $y = x^2$, we get

$$\begin{aligned} x^4 - 6x^2 - 27 &= y^2 - 6y - 27 \\ &= (y - 9)(y + 3) \\ &= (x^2 - 9)(x^2 + 3) \\ &= (x - 3)(x + 3)(x^2 + 3) \end{aligned}$$

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Factor.

1. $16a^4 - 81b^8$

2. $\frac{9n^2}{m^{12}} - \frac{49w^4}{p^6}$

3. $64x^3y^6 + 27z^9$

4. $\frac{1}{8}a^9b^3 - \frac{1}{27}c^{15}$

5. $y^4 - 16y^2 + 63$

6. $n^6 + 5n^3 - 6$

7. $6x^4 + 31x^2 + 35$

8. $15a^6 + 31a^3 - 24$

9. $8x^{-2} + 6x^{-1} - 27$

10. $20y^{-2} - y^{-1} - 30$

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Answers

1. $(2a - 3b^2)(2a + 3b^2)(4a^2 + 9b^4)$

2. $\left(\frac{3n}{m^6} - \frac{7w^2}{p^3}\right)\left(\frac{3n}{m^6} + \frac{7w^2}{p^3}\right)$

3. $(4xy^2 + 3z^3)(16x^2y^4 - 12xy^2z^3 + 9z^6)$

4. $\left(\frac{1}{2}a^3b - \frac{1}{3}c^5\right)\left(\frac{1}{4}a^6b^2 + \frac{1}{6}a^3bc^5 + \frac{1}{9}c^{10}\right)$

5. $(y^2 - 7)(y - 3)(y + 3)$

6. $(n - 1)(n^3 + 6)(n^2 + n + 1)$

7. $(2x^2 + 7)(3x^2 + 5)$

8. $(5a^3 - 3)(3a^3 + 8)$

9. $(2x^{-1} - 3)(4x^{-1} + 9)$

10. $(5y^{-1} + 6)(4y^{-1} - 5)$

Prepared by: Dr. Teresa V. Sutcliffe, Winter 2010