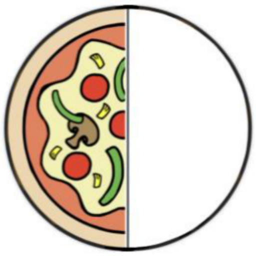
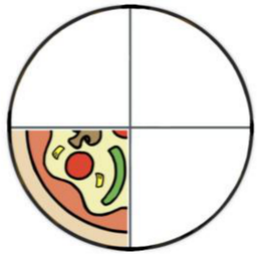
INTRODUCTION TO FRACTIONS

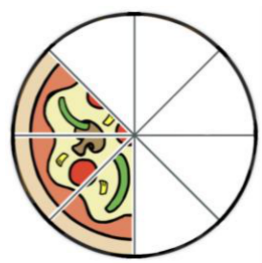
MEANING AND PROPERTIES OF FRACTIONS

Fractions are used to represent parts of a whole.

Example: What is the fraction of the shaded area?







$\frac{1}{2}$ one-half

$\frac{1}{4}$ one-quarter

$\frac{3}{8}$ three-eighths

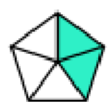
The top number, called the **numerator**, says how many slices we have.
The bottom number, called the **denominator**, says how many equal slices the whole pizza was cut into.

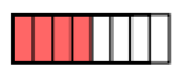
EXERCISES:

1. What is the fraction of the shaded area?

a) 

b) 

c) 

d) 

e) 

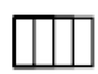
2. Shade the figure that represents the given fraction.

a)  $\frac{3}{8}$

b)  $\frac{2}{3}$

c)  $\frac{4}{5}$

d)  $\frac{2}{5}$

e)  $\frac{2}{4}$

FACTORS

Factors are the numbers we multiply to get another number.

Example: List all factors of 24.

1×24, 2×12, 3×8, 4×6
The factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24.

EXERCISES:

3. List all factors of the following numbers.

a) 12 b) 80 c) 36 d) 45 e) 99

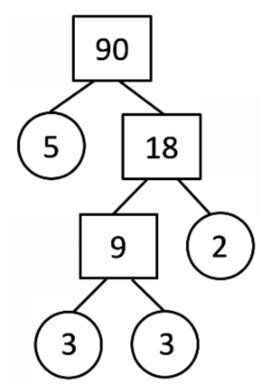
PRIME FACTORIZATION

A **prime number** is a whole number greater than 1 whose factors are only 1 and itself.
Prime numbers = {2, 3, 5, 7, 11, 13, ...}

Prime factorization of a number is to factor a number completely until only prime numbers are left. A factor tree is helpful when finding the prime factorization.

To find the prime factorization:

- Start with any two factors of the number.
- Keep factoring until all branches end with prime numbers.
- The prime factorization is the product of prime numbers.



Example: Find the prime factorization of 90.

From the factor tree to the right, we see that at the end of each branch we have: $90 = \boxed{2 \cdot 3^2 \cdot 5}$

EXERCISES:

4. Find the prime factorization of the following numbers:

a) 75 b) 66 c) 40 d) 81 e) 54

REDUCING A FRACTION TO LOWEST TERMS

To reduce a fraction, we look for common factors between the numerator and denominator. We divide the numerator AND denominator by the largest factor in common, also known as greatest common factor (GCF). Alternatively, we can divide the numerator and denominator by any common factor until there's no more common factor.

Example: Reduce $\frac{12}{32}$ to its lowest term.

Notice that 12 and 32 both have a GCF of 4, so to reduce the fraction we will divide the numerator and the denominator by 4.

$$\frac{12}{32} = \frac{12 \div 4}{32 \div 4} = \frac{3}{8}$$

EXERCISES:

5. Reduce each fraction to lowest terms.

a) $\frac{8}{18}$ b) $\frac{10}{35}$ c) $\frac{16}{24}$ d) $\frac{12}{48}$ e) $\frac{49}{70}$

MULTIPLYING FRACTIONS

When multiplying fractions, we first cancel any common factors between any numerators and denominators. Then we multiply straight across.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Example: Multiply. Simplify, if possible.

a) $\frac{5}{2} \cdot \frac{7}{3}$

Since both numerators don't have a common factor with any denominators, multiply across.

$$\frac{5}{2} \cdot \frac{7}{3} = \frac{35}{6}$$

b) $\frac{10}{7} \cdot \frac{3}{5}$

Notice that 10 (numerator of first fraction) and 5 (denominator of second fraction) have a common factor: 5.

We will divide both by 5 to reduce before multiplying across.

After reducing those values, there are no more common factors between any numerators and denominators, so we multiply across.

$$\cancel{10 \div 5} \quad 2 \quad \_ \quad 2 \quad 2 \quad \_ \quad 4$$

EXERCISES:

6. Multiply. Simplify, if possible.

a) $\frac{1}{2} \cdot \frac{4}{5}$ b) $\frac{3}{5} \cdot \frac{10}{11}$ c) $\frac{7}{10} \cdot \frac{2}{21}$ d) $\frac{15}{16} \cdot \frac{8}{10}$ e) $\frac{5}{10} \cdot \frac{3}{4}$

DIVIDING FRACTIONS

When dividing fractions, multiply the first fraction by the reciprocal ("flip") of the second fraction.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Example: Divide these fractions. Simplify, if possible.

a) $\frac{1}{2} \div \frac{3}{4}$

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \cdot \frac{4}{3} = \frac{1}{\cancel{2 \div 2}} \cdot \frac{\cancel{4 \div 2}}{3} = \frac{1}{1} \cdot \frac{2}{3} = \frac{2}{3}$$

b) $\frac{3}{5} \div 5$

$$\frac{3}{5} \div \frac{5}{1} = \frac{3}{5} \cdot \frac{1}{5} = \frac{3}{25}$$

EXERCISES:

7. Divide. Simplify, if possible.

a) $\frac{2}{3} \div \frac{5}{6}$ b) $\frac{3}{4} \div \frac{1}{4}$ c) $\frac{5}{8} \div \frac{15}{16}$ d) $6 \div \frac{1}{2}$ e) $\frac{2}{5} \div \frac{1}{6}$

LEAST COMMON DENOMINATOR (LCD), also called LEAST COMMON MULTIPLE (LCM)

A **multiple** of a number is the result of multiplying that number by an integer.

Example: List the multiples of 3.

	3×1	3×2	3×3	3×4	3×5	3×6	…
Multiples	3	6	9	12	15	18	…

The multiples of 3 are 3, 6, 9, 12, 15, 18, …

Example: Find the LCD between $\frac{1}{2}$ and $\frac{2}{3}$

Step 1: List the multiples of each denominator.

Multiples of 2 = 2, 4, 6, 8, 10, 12, 14, …

Multiples of 3 = 3, 6, 9, 12, 15, 18, …

Step2 : The first common multiple is the LCD.

The LCD between $\frac{1}{2}$ and $\frac{2}{3}$ is 6.

Example: Find the LCD between $\frac{1}{12}$ and $\frac{2}{15}$

Step 1: List the multiples of each denominator.

Multiples of 12 = 12, 24, 36, 48, 60, 72, 84, 96, …

Multiples of 15 = 15, 30, 45, 60, 75, 90, …

Step2 : The first common multiple is the LCD.

The LCD between $\frac{1}{12}$ and $\frac{2}{15}$ is 60.

EXERCISES:

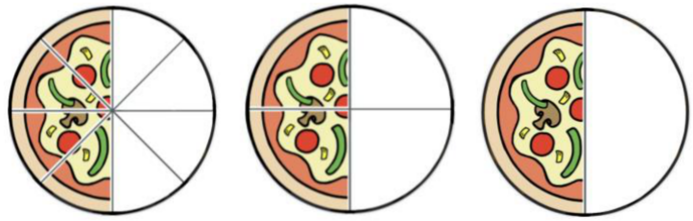
8. Find the least common denominator of each set of fractions.

a) $\frac{2}{5}$; $\frac{2}{3}$ b) $\frac{5}{8}$; $\frac{1}{2}$ c) $\frac{1}{7}$; $\frac{2}{5}$ d) $\frac{3}{4}$; $\frac{1}{2}$ e) $\frac{3}{4}$; $\frac{5}{8}$; $\frac{7}{16}$

LCD = ____ LCD = ____ LCD = ____ LCD = ____ LCD =____

WRITING EQUIVALENT FRACTIONS

Equivalent fractions are fractions that have the same value, but they may look different. To write an equivalent fraction, we either have to multiply or divide the numerator and denominator by the same number, or we can also reduce the fraction to its lowest terms.



$$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$$

Four-eighths Two-quarters One-half

Example: Find the number that belongs in the space by writing an equivalent fraction.

a) $\frac{1}{2} = \frac{\quad}{10}$

$$\frac{1}{2} = \frac{1 \cdot 5}{2 \cdot 5} = \frac{5}{10}$$

b) $\frac{8}{12} = \frac{\quad}{3}$

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

EXERCISES:

9. Find the number that belongs in the space by building or reducing equivalent fractions.

a) $\frac{5}{6} = \frac{\quad}{12}$ b) $\frac{2}{5} = \frac{\quad}{20}$ c) $\frac{7}{14} = \frac{\quad}{2}$ d) $\frac{20}{22} = \frac{\quad}{11}$ e) $\frac{10}{30} = \frac{\quad}{3}$

ADDING AND SUBTRACTING FRACTIONS

To add or subtract fractions, both fractions must have the same denominator. If they do not have the same denominator, find the LCD and write each fraction as an equivalent fraction with the LCD. Once all fractions have the same denominator, add or subtract the numerators and keep the common denominator. Always check if we can reduce further!

Note: A whole number can be written as a fraction. (Example: $3 = \frac{3}{1}$)

Example: Add or subtract.

a) $\frac{3}{5} + \frac{1}{5}$ (same denominator)

$$\frac{3}{5} + \frac{1}{5} = \frac{3+1}{5} = \frac{4}{5}$$

b) $\frac{11}{15} - \frac{1}{3}$ (different denominator; LCD = 15)

$$\frac{11}{15} - \frac{1 \cdot 5}{3 \cdot 5} = \frac{11}{15} - \frac{5}{15} = \frac{11-5}{15} = \frac{6}{15} = \frac{2}{5}$$

Always reduce to lowest terms, if possible.

EXERCISES:

10. Add or subtract.

a) $\frac{2}{7} + \frac{3}{7}$ b) $\frac{5}{8} - \frac{1}{7}$ c) $\frac{5}{4} + \frac{1}{2}$ d) $3 - \frac{3}{7}$ e) $\frac{1}{3} - \frac{1}{2}$

ORDER OF OPERATIONS

$$\text{PE}(\overrightarrow{\text{MD}})(\overrightarrow{\text{AS}})$$

1. **Parenthesis** or other grouping symbols [ ], { }, | |. Start with the innermost parenthesis then work our way towards the outer grouping symbols.
2. **Exponents**
3. **Multiplication OR Division** [LEFT TO RIGHT!]
4. **Addition OR Subtraction** [LEFT TO RIGHT!]

Example: Simplify.

$$\frac{7}{5} \div \left(5 + \frac{1}{2}\right)$$

$$= \frac{7}{5} \div \left(\frac{5 \cdot 2}{1 \cdot 2} + \frac{1}{2}\right) = \frac{7}{5} \div \left(\frac{10}{2} + \frac{1}{2}\right) = \frac{7}{5} \div \frac{11}{2} = \frac{7}{5} \cdot \frac{2}{11} = \frac{14}{55}$$

EXERCISES:

11. Simplify.

a) $\left(\frac{8}{5} + \frac{1}{5}\right) \div 2$

b) $4^2 - \left(\frac{1}{4}\right)^2$

c) $\left(\frac{2}{3} \cdot 5\right) \div \frac{1}{2}$

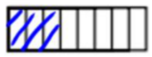
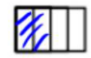
d) $\frac{9}{16} \div \frac{1}{2} \cdot \frac{1}{4}$

e) $\left(\frac{1}{3} - \frac{1}{15}\right)\left(\frac{1}{5} + \frac{1}{3}\right)$

Answers:

1. a. $\frac{3}{5}$ b. $\frac{3}{8}$ c. $\frac{2}{5}$ d. $\frac{5}{7}$ e. $\frac{1}{8}$

2.

a. $\frac{3}{8}$  b. $\frac{2}{3}$  c. $\frac{4}{5}$  d. $\frac{2}{5}$  e. $\frac{2}{4}$ 

3. a. 1, 2, 3, 4, 6, 12
   b. 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
   c. 1, 2, 3, 4, 6, 9, 12, 18, 36
   d. 1, 3, 5, 9, 15, 45
   e. 1, 3, 9, 11, 33, 99

4. a. $75 = 3 \cdot 5^2$ b. $66 = 2 \cdot 3 \cdot 11$ c. $40 = 2^3 \cdot 5$ d. $81 = 3^4$
   e. $54 = 2 \cdot 3^3$

5. a. $\frac{4}{9}$ b. $\frac{2}{7}$ c. $\frac{2}{3}$ d. $\frac{1}{4}$ e. $\frac{7}{10}$

6. a. $\frac{2}{5}$ b. $\frac{6}{11}$ c. $\frac{1}{15}$ d. $\frac{3}{4}$ e. $\frac{3}{8}$

7. a. $\frac{4}{5}$ b. 3 c. $\frac{2}{3}$ d. 12 e. $\frac{12}{5}$

8. a. 15 b. 8 c. 35 d. 4 e. 16

9. a. 10 b. 8 c. 1 d. 10 e. 1

10. a. $\frac{5}{7}$ b. $\frac{27}{56}$ c. $\frac{7}{4}$ d. $\frac{18}{7}$ e. $-\frac{1}{6}$

11. a. $\frac{9}{10}$ b. $\frac{255}{16}$ c. $\frac{20}{3}$ d. $\frac{9}{32}$ e. $\frac{32}{225}$