PROPERTIES OF EXPONENTS

MULTIPLYING POWERS WITH LIKE BASES

An **exponent** indicates how many times the base is a factor. In the expression 2^3, the base is 2 and the exponent is 3. The exponent is indicating that the base 2 is a factor 3 times, that is $2 \cdot 2 \cdot 2$.

The expression $x^4 \cdot x^3$ can be expanded and simplified in the following way

$$x^4 \cdot x^3 = x \cdot x \cdot x \cdot x \;\cdot\; x \cdot x \cdot x = x^7$$

x^4 has four factors of x and is being multiplied to x^3 which has three factors of x, so there is a total of seven factors of x.

PRODUCT OF POWERS

When multiplying two powers with the same base, add the exponents.

$$\boldsymbol{x^m \cdot x^n = x^{m+n}}$$

Examples: Simplify.

a) $x^{12} \cdot x^3$
$= x^{12+3}$
$= x^{15}$

b) $(2^{17} \cdot y^4)(2^{13} \cdot y^9)$
$= 2^{17+13} \cdot y^{4+9}$
$= 2^{30} \cdot y^{13}$

c) $(x + y)^7\,(x + y)^2$
$= (x + y)^{7+2}$
$= (x + y)^9$

d) $b^{5/3} \cdot b^{1/3}$
$= b^{\left(\frac{5}{3}+\frac{1}{3}\right)}$
$= b^{6/3}$
$= b^2$

e) $(xy^6)(3x^2y^7)$
$= 3 \cdot x \cdot x^2 \cdot y^6 \cdot y^7$
$= 3 \cdot x^{1+2} \cdot y^{6+7}$
$= 3x^3y^{13}$

EXERCISES:

(1) y^4y^6

(2) $(2x^3y^4)\,(3x^5y^8)$

(3) $x^{3/5}\,x^{2/5}$

DIVIDING POWERS WITH LIKE BASES

The expression $\frac{x^5}{x^3}$ can be expanded and simplified in the following way:

$$\frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} = \frac{\cancel{x \cdot x \cdot x} \cdot x \cdot x}{\cancel{x \cdot x \cdot x}} = \frac{x \cdot x}{1} = x^2$$

Note that the exponent of the result x^2 is the difference between the exponents of x^5 and x^3.

QUOTIENT OF POWERS

When dividing two powers with the same base, subtract the exponents.

$$\boxed{\frac{x^m}{x^n} = x^{m-n}}$$

Examples: Simplify.

a) $\frac{x^{12}}{x^3}$
$= x^{12-3}$
$= x^9$

b) $\frac{2^{17}y^9}{2^{13}y^4}$
$= 2^{17-13} \cdot y^{9-4}$
$= 2^4y^5$
$= 16y^5$

c) $\frac{(x+y)^7}{(x+y)^2}$
$= (x+y)^{7-2}$
$= (x+y)^5$

d) $\frac{b^{4/3}}{b^{1/3}}$
$= b^{\left(\frac{4}{3}-\frac{1}{3}\right)}$
$= b^{3/3}$
$= b$

e) $\frac{3^2x^8y^{12}}{3x^2y^7}$
$= 3^{2-1} \cdot x^{8-2} \cdot y^{12-7}$
$= 3x^6y^5$

EXERCISES:

(4) $\frac{2^7x^{10}y^{15}}{2x^5y^7}$

(5) $\frac{t^3t^9}{t^4}$

(6) $\frac{(y-8)^9}{(y-8)^5}$

RAISING A POWER TO A POWER

The expression $(x^4)^3$ can be expanded and simplified in the following way:

$$(x^4)^3 = x^4 \cdot x^4 \cdot x^4 = x^{4+4+4} = x^{12}$$

Notice that the exponent of the result $(x^4)^3$ is the product of the powers 4 and 3.

POWER OF A POWER

When dividing two powers with the same base, subtract the exponents.

$$(x^m)^n = x^{m \cdot n}$$

***Example*:** Simplify.

a) $(x^{12})^3$

$= x^{12 \cdot 3}$

$= x^{36}$

b) $(y^7)^2$

$= y^{7 \cdot 2}$

$= y^{14}$

c) $\left(b^{5/3}\right)^3$

$= b^{\left(\frac{5}{3}\cdot\frac{3}{1}\right)}$

$= b^{\left(\frac{5}{\cancel{3}}\cdot\frac{\cancel{3}}{1}\right)}$

$= b^5$

EXERCISES:

(7) $(x^3)^4$

(8) $(x^4)^6(x^2)^3$

(9) $\left(z^{1/3}\right)^{6/5}$

RAISING A PRODUCT OR QUOTIENT TO A POWER

The expression $(2x^4y)^3$ can be expanded and simplified the following way:

$$\begin{aligned}(2x^4y)^3 &= 2x^4y \cdot 2x^4y \cdot 2x^4y \\ &= 2 \cdot 2 \cdot 2 \cdot x^4 \cdot x^4 \cdot x^4 \cdot y \cdot y \cdot y \\ &= 2^{1+1+1} \cdot x^{4+4+4} \cdot y^{1+1+1} \\ &= 2^3 \cdot x^{12} \cdot y^3 \\ &= 8x^{12}y^3\end{aligned}$$

The factors of the product are $2, x^4$, and y. Notice that each factor was cubed, that is

$$(2x^4y)^3 = 2^3 \cdot (x^4)^3 \cdot y^3 = 8x^{12}y^3$$

POWER OF A PRODUCT

When dividing two powers with the same base, subtract the exponents.

$$\boldsymbol{(xy)^n = x^n y^n}$$

The expression $\left(\frac{x^3}{y^2}\right)^3$ can be expanded and simplified the following way:

$$\begin{aligned}\left(\frac{x^3}{y^2}\right)^3 &= \frac{x^3}{y^2} \cdot \frac{x^3}{y^2} \cdot \frac{x^3}{y^2} \\ &= \frac{x^3 \cdot x^3 \cdot x^3}{y^2 \cdot y^2 \cdot y^2} \\ &= \frac{x^9}{y^6}\end{aligned}$$

Notice the numerator x^3 was raised to the third power, that is $(x^3)^3 = x^9$ and the denominator y^2 was also raised to the third power, $(y^2)^3 = y^6$.

POWER OF A QUOTIENT

When raising a quotient to a power, raise the numerator to the power and divide by the denominator to the power.

$$\boldsymbol{\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}}$$

Example: Simplify.

a) $(xy)^3$

$= x^3 \cdot y^3$

$= x^3y^3$

b) $(2^8y^4)^6$

$= 2^{8\cdot 6} \cdot y^{4\cdot 6}$

$= 2^{54} \cdot y^{24}$

c) $\left(\frac{x}{y}\right)^4$

$= \frac{x^4}{y^4}$

d) $\left(\frac{2x}{y^4}\right)^3$

$= \frac{2^3\,x^3}{(y^4)^3}$

$= \frac{8x^3}{y^{12}}$

EXERCISES:

(10) $\left(\frac{c}{d^8}\right)^5$

(11) $\frac{(3x^4y)^3}{x^5}$

(12) $\frac{(6x)^5}{(6x)^3}$

EXPONENTS OF 0 AND 1

THE EXPONENT ONE

For any base x,

$$\boldsymbol{x^1 = x}$$

THE EXPONENT ZERO

A nonzero base raised to the 0 power is 1. For any nonzero base x,

$$\boldsymbol{x^0 = 1}$$

Example: Simplify.

a) $(x+2)^1$
$= x + 2$

b) 3^0
$= 1$

c) $2(4x)^0$
$= 2 \cdot 1$
$= 2$

EXERCISES:

(13) y^0

(14) $(xy)^1(xy)^0$

NEGATIVE EXPONENTS

For any real number x that is nonzero and any integer n,

$$\boldsymbol{x^{-n} = \frac{1}{x^n}} \quad \textbf{and} \quad \boldsymbol{\frac{1}{x^{-n}} = x^n}$$

For any nonzero real numbers x and y and any integer n,

$$\boldsymbol{\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n}$$

where $x, y \neq 0$

Example: Simplify.

a) x^{-3}

$= \frac{1}{x^3}$

b) $2x^{-4}$

$= 2 \cdot x^{-4}$

$= 2 \cdot \frac{1}{x^4}$

$= \frac{2}{x^4}$

c) $\frac{3^{-2}}{x^4}$

$= \frac{1}{3^2 \cdot x^4}$

$= \frac{1}{9x^4}$

d) $\frac{1}{2^{-3}}$

$= 2^3$

$= 8$

e) $\frac{3x^{-3}}{x^{-2}}$

$= 3x^{[-3-(-2)]}$

$= 3x^{-1}$

$= \frac{3}{x}$

f) $\left(\frac{2x^2}{3y^{-3}}\right)^{-4}$

$= \left(\frac{3y^{-3}}{2x^2}\right)^4$

$= \frac{3^4 \cdot y^{-12}}{2^4 \cdot x^8}$

$= \frac{81}{16x^8y^{12}}$

EXERCISES:

(15) $x(y^3 \cdot y^{-3})$

(16) $\frac{5\,t^{-8}}{t^{-3}}$

(17) $(3x^3y)^{-2}$

(18) $\left(\frac{2x^2y^{-5}}{3x^0y^3}\right)^{-3}$

Answers

1.) y^{10}
2.) $6x^8y^{12}$
3.) x
4.) $2^6x^5y^8$
5.) t^8
6.) $(y-8)^4$
7.) x^{12}
8.) x^{30}
9.) $z^{2/5}$
10.) $\frac{c^5}{d^{40}}$
11.) $27x^7y^3$
12.) $36x^2$
13.) 1
14.) xy
15.) x
16.) $\frac{5}{t^5}$
17.) $\frac{1}{9x^6y^2}$
18.) $\frac{27y^{24}}{8x^6}$