

## EXPONENTS, ORDER OF OPERATIONS, AND SQUARE ROOTS.

### EXPONENTIAL NOTATION

Exponential notation is used for repeated multiplication. A product like  $10 \cdot 10 \cdot 10 \cdot 10$  where the factors are the same, is called a power. Powers are often written in exponential notation.

For  $10 \cdot 10 \cdot 10 \cdot 10$  we write  $10^4$  and it is computed to be equivalent to 10,000. The number 10 is called the **base** and the number 4 in this case is called the **exponent**. And the entire expression  $10^4$  is called an **exponential expression**

Example: Write exponential notation for  $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

Solution: The exponential notation is  $4^5$

Example: Simplify (a)  $3^4$  (b)  $-3^4$  (c)  $(-3)^4$  (d)  $(2x)^3$

Solution:

$$(a) 3^4 = 3 * 3 * 3 * 3 = 81$$

$$(b) -3^4 = -3 * 3 * 3 * 3 = -81$$

$$(c) (-3)^4 = (-3) * (-3) * (-3) * (-3) = 81$$

$$(d) (2x)^3 = (2x)(2x)(2x) = 2 * 2 * 2 * x * x * x = 8x^3$$

### EXPONENTIAL NOTATION

For any natural number  $n$ ,  $b^n$  means  $b * b * b * \dots * b$  where there are  $n$  factors of  $b$ .

Note:  $5^0 = 1$ . In fact  $N^0 = 1$  when  $N$  is any number other than 0.

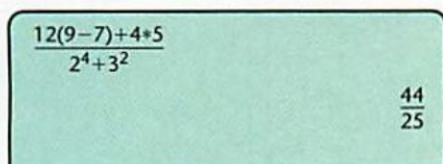
## Technology Connection

To enter an exponential expression on a graphing calculator, we press  $\wedge$  and then enter the exponent. The  $x^2$  key can be used to enter an exponent of 2.

The following screen shows the calculation

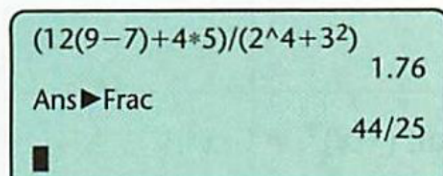
$$\frac{12(9 - 7) + 4 \cdot 5}{2^4 + 3^2}$$

from Example 8 using a calculator with an n/d option.



A calculator screen with a light green background. On the left, the expression  $\frac{12(9-7)+4*5}{2^4+3^2}$  is displayed. On the right, the result  $\frac{44}{25}$  is shown.

On calculators without an n/d option, we use parentheses to enclose the numerator and the denominator. The following screen shows the above calculation using parentheses. The number has also been converted to fraction notation.



A calculator screen with a light green background. At the top, the expression  $(12(9-7)+4*5)/(2^4+3^2)$  is displayed. Below it, the decimal result 1.76 is shown. At the bottom, the text "Ans ▶ Frac" is displayed, followed by the fraction result 44/25. A small black square is visible in the bottom left corner.

## ORDER OF OPERATIONS

How do we compute  $6 \div 3 \times 2$ ? There is a standard order in which we compute more complicated expressions. It is called the rules for order of operations.

### RULES FOR ORDER OF OPERATIONS

1. Simplify, if possible, within the innermost grouping symbols,  $( )$ ,  $\{ \}$ ,  $[ ]$ ,  $| |$ , and above or below any fraction bars.
2. Simplify all exponential expressions.
3. Perform all multiplications and divisions, working from left to right.
4. Perform all additions and subtractions, working from left to right.

Example: Simplify  $6 \div 3 \times 2$

$$\text{Solution: } 6 \div 3 \times 2 = 2 \times 2 = 4$$

Example: Simplify  $2 \cdot 4 - 12 \div 3$

$$\begin{aligned}\text{Solution: } 2 \cdot 4 - 12 \div 3 \\ &= 8 - 12 \div 3 \\ &= 8 - 4 \\ &= 4\end{aligned}$$

Example: Simplify  $6^2 \div 3 * 4$

$$\begin{aligned}\text{Solution: } 6^2 \div 3 * 4 \\ &= 36 \div 3 * 4 \\ &= 12 * 4 \\ &= 48\end{aligned}$$

Example: Simplify  $(12 - 8)^3 + 2^4 \div 4$

$$\begin{aligned}\text{Solution: } (12 - 8)^3 + 2^4 \div 4 \\ &= 4^3 + 2^4 \div 4 \\ &= 64 + 16 \div 4 \\ &= 64 + 4 \\ &= 68\end{aligned}$$

Example: Simplify  $5^2 + [3^2 - (10 \div 2)] - 9 * 3$

$$\begin{aligned}\text{Solution: } 5^2 + [3^2 - (10 \div 2)] - 9 * 3 \\ &= 5^2 + [3^2 - 5] - 9 * 3 \\ &= 5^2 + [9 - 5] - 9 * 3 \\ &= 5^2 + 4 - 9 * 3 \\ &= 25 + 4 - 9 * 3 \\ &= 25 + 4 - 27 \\ &= 29 - 27 \\ &= 2\end{aligned}$$

Example: Simplify  $\frac{7-2*3+3^2}{5(2-1)}$

Solution:  $\frac{7-2*3+3^2}{5(2-1)}$   
 $= \frac{7-2*3+9}{5(1)}$   
 $= \frac{7-6+9}{5}$   
 $= \frac{1+9}{5}$   
 $= \frac{10}{5}$   
 $= 2$

Example: Simplify  $24 \div 8 * 3 + 6$

Solution:  $24 \div 8 * 3 + 6$   
 $= 3 * 3 + 6$   
 $= 9 + 6$   
 $= 15$

Practice Set:

Write using exponential notation

1.  $3 * 3 * 3 * 3$
2.  $2 * x * x * x$
3.  $(3n) * (3n) * (3n) * (3n)$
4.  $4 * 5 * 5 * 5 * 5 * 5$
5.  $7 * 7 * 2 * 11 * 11 * 11 * 11$

Evaluate:

1.  $6^3$
2.  $2^7$
3.  $20^1$
4.  $3 * 2^6$
5.  $2 * 7^2$
- 6.

Simplify:

1.  $14 \div 7 * 2 + 4$
2.  $12 + 6 * 4$
3.  $50 \div 5 * 5 + 8$
4.  $42 \div 7 * 2$
5.  $36 + \frac{12}{4} - 18 \div 3 + 2$
6.  $5^3 \div (10 + 15) + 9^2 + 3^3$
7.  $18 - 7 \div 0$
8.  $(12 - 3 * 2^2) \div 15$
9.  $4^4 * 6 - (100 \div 10)$
10.  $[40 - (10 - 3)] - 2^5$
11.  $(5 * 6) + [9 \div (18 \div 6)]$
12.  $(24 \div 6) + [(5 - 3) * 2]$
13.  $35 \div [3^2 + (9 - 7) - 2^2] + 10 * 3$
14.  $\frac{9^2 + 2^2 - 1^2}{8 \div 2 * 3 \div 3}$
15.  $27 - [5 + 3[8 * (10 - 8)]] - 50$
16.  $5(-9) + 3$
17.  $-12 + 6 \div 2$
18.  $5 + 9 * 4 - 34$
19.  $\frac{23 - (-4)}{-1}$
20.  $\frac{51}{18 \div 3 \div 2}$
21.  $7 * 5^2 - (-3)^3$
22.  $7 - (-11)^2$
23.  $10 \cdot 2^4 - 12 \cdot (-3)^3$
24.  $|12 - 19| \div 7$
25.  $-(-7)^3$
26.  $(2 - 9)^2 \div (18 - 11)^1$
27.  $|9 - 13| \cdot 4^3 \div (-2)$
28.  $12 - [7 - (3 - 6)] + (2 - 3)^3$
29.  $\frac{(-7)(-3) - (4)(3)}{3[7 \div (3 - 10)]}$
30.  $-2[6 + 4(2 - 8)] - 25$
31.  $\frac{1}{2} + \frac{1}{6} * \frac{1}{3}$

$$32. 2^2 - \left(\frac{1}{3}\right)^2$$

$$33. 3^2 - \left(\frac{1}{2}\right)^2$$

$$34. \left(\frac{2}{9} + \frac{4}{9}\right) + \left(\frac{1}{3} - \frac{9}{10}\right)$$

$$35. \left(-\frac{2}{3} - \frac{7}{3}\right) \div \frac{4}{9} - 2$$

$$36. \left(\frac{8}{9}\right) \div \left(2 - \frac{2}{3}\right)^2$$

$$37. \left(\frac{3}{4} + \frac{1}{8}\right)^2 - \left(\frac{1}{2} + \frac{1}{8}\right)$$

$$38. \frac{3 - \frac{1}{2}}{4 + \frac{2}{5}}$$

$$39. 2 \div 12 \cdot 2 + 4$$

$$40. (-3)^2 \cdot 6 \div (7 - 9)^3 - 2[1 - 5((2 - 6) + 4^2)]$$

$$41. \frac{(-5)^2 - 3 \cdot 5}{3^2 + 4[6 - 7] * (-1)^7}$$