

# GRAPHING LINEAR EQUATIONS

**STANDARD FORM**  
 $Ax + By = C$

**SLOPE-INTERCEPT FORM**  
 $y = mx + b$

**POINT-SLOPE FORM**  
 $y - y_1 = m(x - x_1)$

## METHOD 1: Create a Table of Values

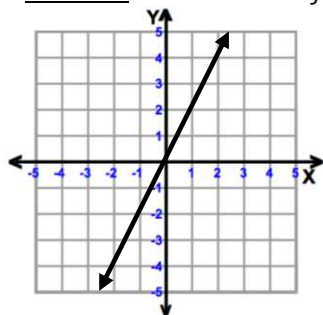
1. Write the equation in slope-intercept  $y = mx + b$  form.
2. Select a value for one coordinate and calculate the corresponding value of the other coordinate.
3. Repeat step (1) to find a second ordered pair. A third ordered pair can be used as a check.
4. Plot the ordered pairs and draw a straight line passing through the points. The line represents all solutions of the equation.

## METHOD 2: Use the Slope and Y-intercept

1. Write the equation in slope-intercept  $y = mx + b$  form.
2. Plot the y-intercept on the graph. This y-intercept coordinate is  $(0, b)$ .
3. From this point, use the slope to find a second point. The slope  $m$  is  $\frac{\text{rise}}{\text{run}}$ .
4. Draw the line that connects the two points.

**Example:**  $y = 2x + 1$

Method 1: Create a table of values



x	y
0	1
1	3
2	5

$$y = 2(0) + 1$$

$$y = 2(1) + 1$$

$$y = 2(2) + 1$$

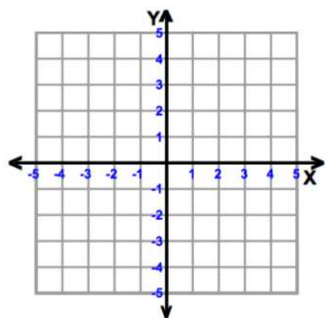
Method 2: Use the slope and y-intercept

The y-intercept is  $(0, 1)$ . The slope is 2, so it can be written as  $\frac{2}{1}$ . From the y-intercept, use the slope of  $\frac{\text{up } 2}{\text{right } 1}$  to plot the second point.

# FINDING THE EQUATION OF THE LINE

**Example 1: (Slope-intercept form) Graph.**

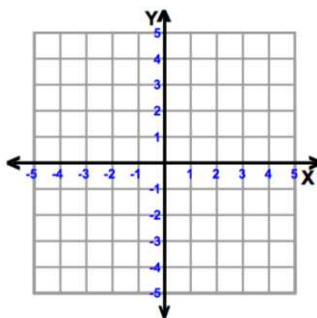
a) Graph  $y = x + 1$



x	y

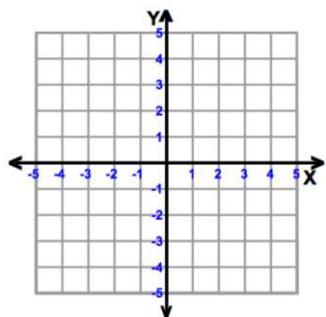
**Example 2: (Standard form) Graph.**

a) Graph  $2x - y = 3$



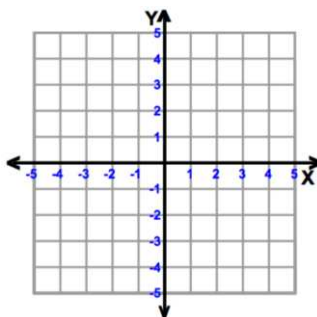
x	y

b) Graph  $y = \frac{5}{3}x - 2$



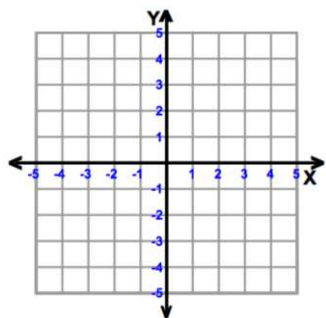
x	y

b) Graph  $x + 3y = 4$



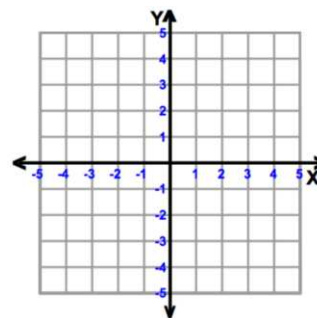
x	y

c) Graph  $y = -\frac{1}{2}x + 1$



x	y

c) Graph  $-2x + 3y = -3$



x	y

**SLOPE-INTERCEPT FORM**

$$y = mx + b$$

**POINT-SLOPE FORM**

$$y - y_1 = m(x - x_1)$$

**Example 1:** Find an equation of the line with slope  $\frac{1}{5}$  and point (0, 7).

The coordinate (0, 7) is a y-intercept, so  $b = 7$ .

$$y = mx + b$$

$$y = \frac{1}{5}x + 7$$

**Example 2:** Find an equation of the line with slope  $\frac{3}{2}$  and point (6, 0).

Method 1: Use the slope-intercept form.

$$y = mx + b$$

$$y = \frac{3}{2}x + b (*)$$

$$0 = \frac{3}{2}(6) + b$$

$$0 = 9 + b$$

$$-9 = b$$

Update (\*).

$$y = \frac{3}{2}x - 9$$

Method 2: Use the point-slope form.

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{3}{2}(x - 6)$$

$$y = \frac{3}{2}x - 9$$

**Example 3:** Find an equation of the line with slope 4 and point (2, 5).

Method 1: Use the slope-intercept form.

$$y = mx + b$$

$$y = 4x + b (**)$$

$$5 = 4(2) + b$$

$$5 = 8 + b$$

$$-3 = b$$

Update (\*\*).

$$y = 4x - 3$$

Method 2: Use the point-slope form.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 4(x - 2)$$

$$y - 5 = 4x - 8$$

$$y = 4x - 3$$

**Example 4:** Find an equation of the line containing the points (3, 2) and (9, -6).

First, find the slope using the formula  $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{-6 - 2}{9 - 3} = \frac{-8}{6} = \frac{-4}{3}$$

Method 1: Use the slope-intercept form.

$$y = mx + b$$

$$y = \frac{-4}{3}x + b (***)$$

$$2 = \frac{-4}{3}(3) + b$$

$$2 = -4 + b$$

$$6 = b$$

Update (\*\*\*)

$$y = \frac{-4}{3}x + 6$$

Method 2: Use the point-slope form.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-4}{3}(x - 3)$$

$$y - 2 = \frac{-4}{3}x + 4$$

$$y = \frac{-4}{3}x + 6$$

**Example 1:** Find an equation of the line given the slope and point.

a) slope  $\frac{2}{3}$  and point (0, 1)

b) slope  $\frac{-7}{2}$  and point (0, -4)

Equation: \_\_\_\_\_

Equation: \_\_\_\_\_

# INTERPRET THE SLOPE AND Y-INTERPRET

c) slope  $\frac{5}{4}$  and point  $(-8, 0)$

Equation: \_\_\_\_\_

d) slope  $-3$  and point  $(2, 0)$

Equation: \_\_\_\_\_

e) slope  $-\frac{1}{5}$  and point  $(5, 7)$

Equation: \_\_\_\_\_

f) slope  $1$  and point  $(3, 6)$

Equation: \_\_\_\_\_

**Example 2:** Find an equation of the line containing the two points.

a)  $(1, 2)$  and  $(3, -8)$

Equation: \_\_\_\_\_

b)  $(2, -3)$  and  $(4, -2)$

Equation: \_\_\_\_\_

## Interpret the slope and y-intercept

**Example 1:** When you have a physical exam, your doctor draws blood for your cholesterol test. Your cholesterol count is measured in milligrams per deciliter (mg/dL). A woman's total cholesterol  $y$  is related to her age  $x$  by the following linear equation:

$$y = 1.1x + 157$$

- a) Determine and interpret the slope of the equation.

The slope is  $\frac{1.1\text{mg/dL}}{1\text{ year}}$

Interpretation: The total cholesterol of a female increases by 1.1 mg/dL as age increases by 1 year.

- b) Determine and interpret the y-intercept of the equation.

The y-intercept is (0, 157).

The total cholesterol of a newborn girl is 157 mg/dL.

- c) Estimate the total cholesterol of a female at age 30. Interpret this in a complete sentence.

$$y = 1.1(30) + 157 = 190$$

The total cholesterol of a 30 year old woman is 190 mg/dL.

**Example 2:** The temperature dropped rapidly overnight. Starting at 80°F at midnight, the temperature dropped 3°F per minute. The temperature  $T$  is related to the number of minutes  $x$  can be represented by the following linear equation:

$$T = -3x + 80$$

- a) Determine and interpret the slope of the equation.

The slope is  $-3$ .

Interpretation: The temperature decreases by 3°F as time increases by 1 minute.

- b) Determine and interpret the y-intercept of the equation.

The y-intercept is (0, 80).

The temperature is 80°F at midnight.

- c) Estimate the temperature when it is 12:10am. Interpret this in a complete sentence.

$$T = -3(10) + 80 = 50$$

The temperature is 50°F at 12:10am.

**Example 1:** Some costs involved in owning a car are affected by the number of miles driven (gas and maintenance) Suppose the annual cost  $y$  of operating a Toyota Camry is related to the number of miles driven  $x$ . The annual cost of operating a Toyota Camry is \$0.25 per mile plus \$2000 by the following linear equation:

$$y = 0.25x + 2000$$

- a) Determine and interpret the slope of the equation.

The slope is  $\frac{\$0.25}{1\text{ mile}}$

Interpretation: The annual cost of owning a Camry increases by \$0.25/mile as the mileage increases by 1 mile.

- b) Determine and interpret the y-intercept of the equation.

The y-intercept is (0, 2000).

Interpretation: The annual cost of a Toyota Camry is \$2000 when 0 miles are driven.

- c) Estimate the annual cost of a Toyota Camry when 500 miles are driven. Interpret this in a complete sentence.

$$y = 0.25(500) + 2000 = 2125$$

Interpretation: The annual cost of a Toyota Camry is \$2125 when 500 miles are driven.

**Example 2:** The cost per minute of talk time for cell phone users has gone down over the years. In 1995, cell phone users paid, on the average, \$0.56 per minute. In 2011, they paid \$0.05 per minute. Assuming that the rate of decline of the cost per minute was constant, the cost per minute can be calculated by the equation

$y = -0.031875x + 0.56$ , where  $x$  represents the number of years after 1995 and  $y$  represents the cost per minute of cell phone usage in dollars.

$$y = -0.031875x + 0.56$$

- a) Determine and interpret the slope of the equation.

The slope is  $-0.031875$ .

Interpretation: The cost per minute of cell phone usage decreases by \$0.03 per year.

- b) Determine and interpret the y-intercept of the equation.

The y-intercept is (0, 0.56).

Interpretation: The cost per minute of cell phone usage is \$0.56 in year 1995.

- c) Estimate the cost per minute of cell phone usage in year 2003. Interpret this in a complete sentence.

$$y = -0.031875(8) + 0.56 = 0.305$$

Interpretation: The cost per minute of cell phone usage is \$0.31 in year 2003.

**Example 3:** A teen magazine began with a circulation of 500,000 prints in its first year. Since then, the circulation has increased an average of 33,388 per year. The equation  $y = 33,388x + 500,000$  represents the number of prints  $y$  and the number of years since its initial production  $x$ .

- a) Determine and interpret the slope of the equation.

- b) Determine and interpret the y-intercept of the equation.

- c) Estimate the number of prints after the teen magazine's seventh year. Interpret this in a complete sentence.